Generation Of Modulus Constraint Signal In Adaptive Radar Waveform

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Abstract— Waveform synthesis techniques based on maximization of signal-to-interference-noise ratio (SINR) or mutual information (MI) requires generation of transmit signal with a prescribed Fourier magnitude spectrum which is dependent on the varying target and clutter environment conditions. For all practical radar purposes the Power Amplifier is operated at saturation, thus a constant envelope waveform is required to be transmitted. In this paper we consider the problem of generating a signal with constant envelope in time domain from its SINR/SNR maximizing Fourier magnitude spectrum. Alternating projection methods on non-convex sets suffer from poor convergence, local maxima and sensitivity to initial seed. We propose here the application of Zadoff-Chu sequence as the starting point to the iterative alternating projection method. Simulation results that demonstrate the efficiency of this algorithm are presented.

Keywords— Waveform Synthesis, Constant Modulus Constraint, Matched Illumination, Extended Radar Targets, Alternating Projection Methods.

I. INTRODUCTION

With the advent of adaptive transmit waveform design in radar and sonar, the problem of reconstructing constant envelope signals with prescribed Fourier transform has become of prime importance. A radar system is considered adaptive if one or more of its transmit and/or receive parameters are altered based on its operating environment [3]. The adaptation of transmit waveform has derived considerable interest in recent times owing to the considerable performance benefit that it can provide. For maximum efficiency the power amplifiers of the radar is usually operated at saturation, thus demanding a constant envelope time signal. Hence the reconstruction of constant envelope signal from adaptive radar transmit optimized waveform becomes a mandate. The design of radar optimized waveforms are usually based on criteria of maximization of signal-to-interference noise ratio (SINR) or signal-to-noise ratio (SNR) for target detection n[5][10][8], and maximization of mutual information or Mahalanobis distance for target classification[6][8][11].

Formulating the problem mathematically, we have

$$U_m(f)e^{j\theta(f)} = F\{u_e(t)e^{j\phi(t)}\}$$

(1)

where $F\{.\}$ denotes Fourier transform operation.

The function $u_e(t)e^{j\phi(t)}$ describes the complex modulation of the signal, where $u_e(t)$ specifies the time-envelope of the waveform, which as per radar system constraint is required to be constant, say A. The Fourier transform of $u_e(t)e^{j\phi(t)}$ is denoted as $U_m(f)e^{j\theta(f)}$. Most of the adaptive transmit waveform solutions specify the modulus of the Fourier spectra, $U_m(f)$. The problem of finding $\theta(f)$ and/or $\phi(t)$, which meets both the desired time envelope constraint, $u_e(t)$ and the desired Fourier modulus spectra, $U_m(f)$ is called phase retrieval. The question that arises here is - if it is possible to specify time envelope $u_e(t)$ and Fourier spectra $U_m(f)$ independently, since the Fourier transform operation does seem to pose some constraint on the modulus of its Fourier pairs. However as has been found in literature, as the time-bandwidth product of the signal becomes large the bearing of modulus of its Fourier pairs, $u_e(t)$ and $U_m(f)$, on each other, starts to lose force.

There are several iterative solutions proposed in literature, one of the earliest being Gerchberg-Saxton algorithm (GSA), which has been shown to be a special case of Error Reduction Algorithm (ERA) [7]. Based on ERA framework there are other general algorithms based on steepest-descent/conjugate-gradient that have been proposed in literature. Also, basic input-output (BIO) algorithm and Hybrid input-output (HIO) algorithms have been studied [7].

The alternating projection iterative algorithms usually suffer from slow convergence, convergence stagnation, permutation and scaling ambiguities, and sensitivity to initial seed. We here propose to use Zadoff-Chu sequence as the starting point for these alternating projection algorithms. Other fixed starting points such as Huffman's code [2] has been proposed in literature.

Notation: We denote time domain signal and frequency domain spectra with lower and upper-case respectively. The vectors and matrices are represented by an under-bar and boldface respectively. The superscripts $(.)^*, (.)^T$, and $(.)^H$ represent complex conjugation, transposition and complex transposition respectively. The subscript m and e are used to denote Fourier Transform Magnitude (FTM) constraint and time envelope constraints respectively.

II. PROBLEM FORMULATION

From the Transmit-Receive radar model in Figure 1, the received signal at the receiver can be expressed as

$$y(t) = h(t) * x(t) + c(t) * x(t) + n(t)$$

(2)

where $*$ represents convolution, $h(t)$ is the target impulse response which we consider to be extended in time. Extended target impulse response can arise if the target has an extended range or in case of a high resolution radar. Several models of extended target can be found in [9][5][10]. The target impulse response taps of length N can be assumed to be deterministic or stochastic. The clutter response $c(t)$ is from a dense background and is spread through-out in time, and manifests at the receiver as self-interference term, $x(t) * c(t)$. $n(t)$ represents the receiver thermal noise, and $r(t)$ the receiver filter response.
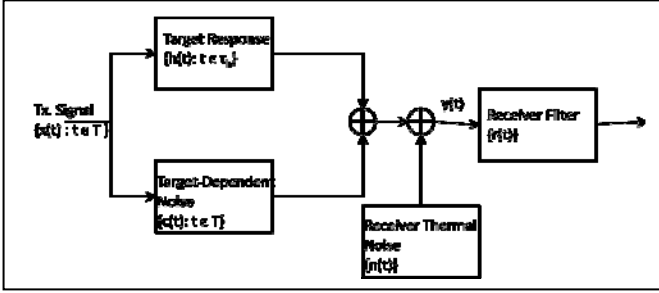

Figure 1 Radar Tx-Rx Model

A. SINR-Matched Illumination

The SINR optimized transmitted signal $\{X_{SINR}(f)\}$ can be computed by solving

$$X_{SINR}(f) = \max_{X(f)} \int_{f \in \Omega} \frac{|H(f)X(f)|^2}{S_{cc}(f)|X(f)|^2 + S_{nn}(f)} df$$

(3)

The solution is

$$|X_{SINR}(f)|^2 = \frac{\sqrt{|H(f)|^2 S_{nn}(f)}}{S_{cc}(f)} \left[\mu - \sqrt{\frac{S_{nn}(f)}{|H(f)|^2}} \right]^+$$

(4)

where μ is the Lagrangian multiplier constant determined from the energy constraint $\int_\Omega |X(f)|^2 df = E$, and $[x]^+ = \max(0, x)$, [6].

A similar $|X_{SINR}(f)|^2$ maximizes the input SINR at the receiver for the extended stochastic targets as well [6]. Note that the self-interference clutter term $\frac{\sqrt{|H(f)|^2 S_{nn}(f)}}{S_{cc}(f)}$ modulates the conventional water-filling solution, $\left[\mu - \sqrt{\frac{S_{nn}(f)}{|H(f)|^2}} \right]^+$.

The energy of the transmitted signal, E is a monotonic function of the Lagrangian multiplier constant, μ. The proof of which is given in Appendix VI.

B. SNR-Matched Illumination

Following equation (3), the signal-to-noise ratio (SNR) maximization problem, in the absence of clutter, becomes

$$X_{SNR}(f) = \max_{X(f)} \int_\Omega \frac{|H(f)|^2}{S_{nn}(f)} |X(f)|^2 df$$

(5)

The solution is given by the eigenfunction of the Freedholm equation of the first kind [8]

$$\lambda_{max} x_{SNR}(t) = \int_T L(t - \tau) x_{SNR}(\tau) d\tau$$

(6)

where the kernel $L(t) = F^{-1}\{\frac{|H(f)|^2}{S_{nn}(f)}\}$.

Now note from equation (5), that only Fourier Transform Magnitude (FTM), $|X_{SNR}(f)|$ is relevant for maximizing SNR.

C. Constant Modulus Waveform

From the adaptive Matched Illumination waveforms (SINR-MI and SNR-MI), the Fourier transform magnitude over a frequency domain is specified. For practicality of the matched illumination waveforms from $|X(f)|^2$, the generation of a constant envelope signal is important. From hereon, we denote the desired Matched Illumination FTMs, i.e. $|X_{SINR}(f)|$ & $|X_{SNR}(f)|$ as $U_m(f)$. Thus, mathematically the problem is

$$x(t) = \begin{cases} A & if \ t \in T \\ 0 & if \ o.w. \end{cases}$$

(7)

suchthat. $|F\{x(t)\} - U_m(f)|^2$ is minimum over $f \in \Omega$.

III. ALTERNATING PROJECTION METHODS

A. Definitions

Developing from the previous section and applying it to our problem here, we define the two sets, D_m and O_e as follows

$$D_m = \{|X(f)| = U_m(f): f \in \Omega\} \qquad (8)$$

i.e. D_m is the set of all waveforms that satisfy the prescribed Fourier Transform Magnitude (FTM) constraint, $U_m(f)$. So essentially the set $D_m(f)$ consists of waveforms of the form $|X(f)|e^{j\theta(f)}$, where $\theta(f) \in R \ \forall \ f \in \Omega$.

$$O_e = \{|x(t)| = u_e(t): t \in T\} \qquad (9)$$

i.e. O_e is the set of all waveforms that satisfy the desired time envelope constraint, $u_e(t)$, which in our case is a constant A. So essentially the set O_e consists of waveforms of the form $|x(t)|e^{j\phi(t)}$, where $\phi(t) \in R \ \forall \ t \in T$.

It is worth noting here that the sets D_m and O_e are closed but not convex, i.e. an iterative projection onto the two sets isn't guaranteed to result in a singleton solution in $\{D_m \cap O_e\}$[2].

Defining two projection operators onto the set D_m and O_e as,

$$\Pi_m\big(X_m(f)e^{j\theta(f)}\big) = U_m(f)e^{j\theta(f)}.1_\Omega$$

(10)

$$\Pi_A\big(x_e(t)e^{j\phi(t)}\big) = Ae^{j\phi(t)}.1_T$$

(11)

where 1_Ω and 1_T are indicator function defined as below

$$1_\Omega(f) = \begin{cases} 1 & if \ f \ \in \Omega \\ 0 & if \ o.w. \end{cases}$$

(12)

$$1_T(t) = \begin{cases} 1 & if \ t \ \in T \\ 0 & if \ o.w. \end{cases}$$

(13)

B. Error Reduction Algorithm Algorithms Applied to Our Problem

The Error Reduction Algorithm (ERA) or Gerchberg-Saxton's algorithm (GSA) [7] requires satisfying the function time-constraint and FTM constraints iteratively by applying minimal changes until convergence. The steps of the algorithm can be outlined as below

1. $X_k(f) = |X_k(f)|e^{j\theta(f)} = F\{x_k(t)\}$

(14)

2. $\tilde{X}_k(f) = \begin{cases} U_m(f)e^{j\theta_k(f)} & if \ f \in \Omega \\ |X_k(f)|e^{j\theta_k(f)} & if \ o.w. \end{cases}$

(15)

3. $x_k(t) = |x_k(t)|e^{j\phi_k(t)} = F^{-1}\{\tilde{X}_k(f)\}$

(16)

4. $\tilde{x}_k(t) = \begin{cases} Ae^{j\phi_k(t)} & if \ t \in T \\ |x_k(t)|e^{j\phi_k(t)} & o.w. \end{cases}$

(17)

5. $x_{k+1}(t) = \tilde{x}_k(t)$

(18)

The steps $1-5$ are iterated until the pre-defined convergence criteria is met. The steps 2 and 4 can be expressed in terms of projection operators as follows

$$\tilde{X}_k(f) = \Pi_m(X_k(f))$$

(19)

$$\tilde{x}_k(t) = \Pi_A(x_k(t))$$

(20)

respectively. And the Normalized Mean-Squared Error (NMSE) at k^{th} iteration in step 2 and 4 are defined as

$$B_k = \int_{f\in\Omega} [U_m(f) - |X_k(f)|]^2 \frac{1}{\Omega} \, df$$

(21)

$$E_k = \int_{t\in T} [A - |x_k(t)|]^2 \frac{1}{T} dt$$

(22)

Remarks: The alternating projection algorithms usually suffer from the following limitations 1) slow convergence, 2) stuck in local maxima/minima, 3) solution stagnation, 4) high computational cost (multiple FFT/IFFT operations), 5) search for step size parameter (Conjugate Gradient, BIO and HIO) and 6) sensitivity to initial seed. The Hybrid Input-Output algorithm though promises to avoid the solution stagnation problem that is usually observed in the other algorithms.

The convergence of these iterative algorithms is sensitive to the intial seed; hence an appropriate choice of initial seed is required to ensure faster convergence and reduced mean squared error.

IV. PROPOSED SOLUTION & RESULTS

Alternatively to [2] where Huffman's signal is proposed to be used as the initial seed, we here propose to use an alternate seed using the Zadoff-Chu sequence[12] as the initial seed to the above alternating projections algorithms. The Zadoff-Chu sequence was verified to have similar performance as reported in [2] with Huffman code. Both Huffman and Zadoff-Chu sequence have faster convergence and lower computation cost (due to reduced iterations to converge), compared to random initial seed.

We present the performance of the proposed solution numerically. Although the proposed initial solution can be used for any alternating projection algorithms of [7] we limit ourselves only to Error Reduction Algorithm. We compare the proposed method of starting with Zadoff Chu sequence with random seed. We use the Normalized Mean Squared Error, B_k of (21) as our metric of comparison, since the time envelope poses a stricter constraint in practical radar. We run both the methods for fixed number of iterations.

The FTM of the following response are used as prescribed FTM,$U_m(f)$, for evaluating the performance of our proposed solution.

$$q_1(t) = e^{-0.03t}\cos(0.2\pi t)$$

(23)

$$q_2(t) = 3e^{-0.03t}\cos(0.4\pi t) + 10e^{-0.32t}\sin(0.4\pi t)$$
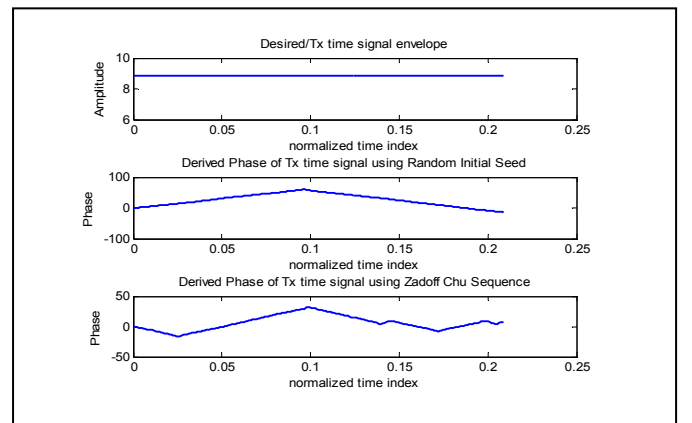
(24)

Figure 2 (top) Transmitted Time Envelope, (mid) Retrieved Phase from Random Initial Seed, (bottom) Retrieved Phase from Zadoff Chu sequence Initial Seed
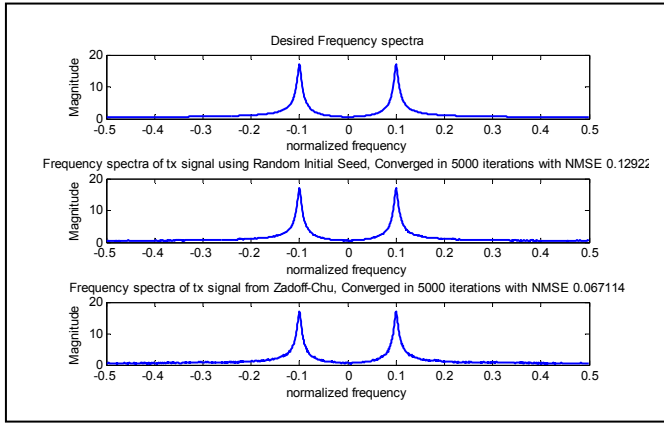
Figure 3 (top) Desired Fourier Transform Magnitude, (mid) Fourier Transform Magnitude from Random Initial Seed, (bottom) Fourier Transform Magnitude from Zadoff Chu GSA
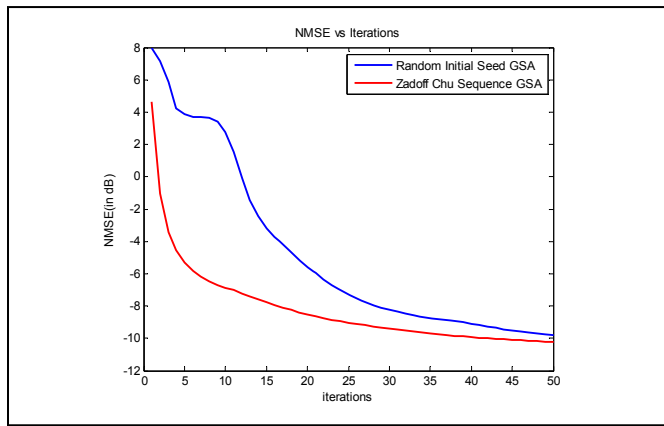

Figure 4 Comparison of convergence of Normalized Mean Square Error with iterations

Figure 2 (top) shows the desired/converged time envelope of the transmit signal, Figure 2 (mid) and Figure 2 (bottom) plots the retrieved temporal phase from the Random Initial Seed and on using Zadoff Chu sequence respectively for $U_m(f) = |F\{q1\}|$. Figure 3 (top) plots the prescribed FTM, i.e. $U_m(f) = |F\{q1\}|$, Figure 3 (mid) and Figure 3 (bottom) compares the corresponding FTM of converged constant-modulus signal from Random Initial Seed and on using Zadoff Chu ssequence as Initial Seed respectively after pre-specified number of iterations. The NMSE of the final solution, i.e. B_{5000} for the Random Initial Seed and Zadoff Chu sequence are 0.12922 and 0.067114 respectively. Figure 4 compares the convergence rate, i.e. NMSE B_k (in dB) vs iterations, between the Random Initial Seed and Zadoff Chu sequence as Initial Seed for $U_m(f) = |F\{q1\}|$.

Figure 5, Figure 6 and Figure 7 compares the temporal response, Fourier Transform Magnitude, and convergence properties for the Random Initial Seed and Zadoff Chu sequence for $U_m(f) = |F\{q2\}|$. The NMSE of the converged solutions are 0.27834 and 0.26783 respectively.

It is also worth noting here that use of Zadoff Chu sequence or Huffman signal as initial seed loosely guarantees the final solution with good auto-correlation properties, but from target detection point of view more important is the

cross-correlation of the target-distorted transmitted signal with the optimal filter response, which actually represents the receiver output of the SINR/SNR-Matched illumination solution. We present the cross-correlation properties of our constant modulus SINR/SNR-Matched Illumination solution elsewhere [13].
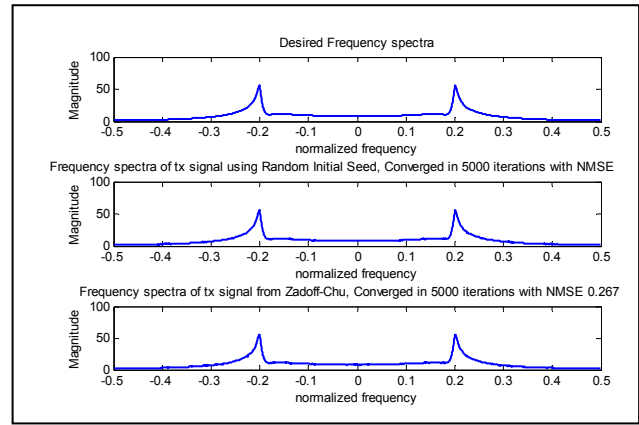

Figure 5 (top) Transmitted Time Envelope, (mid) Retrieved Phase from Random Intial Seed, (bottom) Retrieved Phase from Zadoff Chu GSA
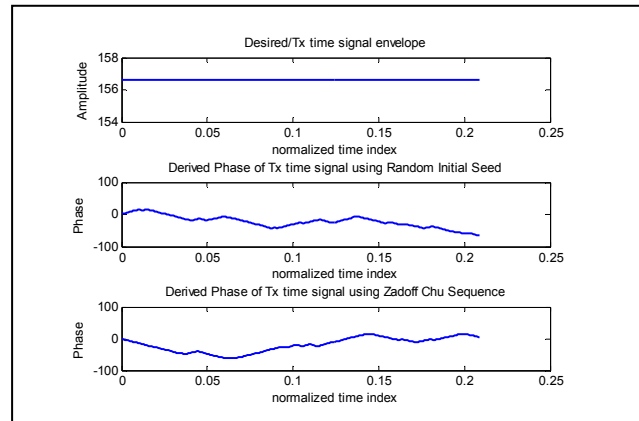

Figure 6 (top) Desired Fourier Transform Magnitude, (mid) Fourier Transform Magnitude from Random Initial Seed, (bottom) Fourier Transform Magnitude from Zadoff-Chu GSA.
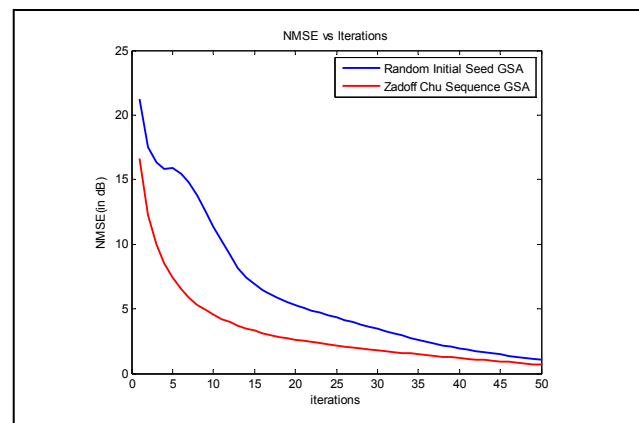

Figure 7 Comparison of convergence of Normalized Mean Square Error with iterations

V. CONCLUSION

In this paper, we have proposed to Zadoff Chu sequence as the starting point for alternating projection algorithms for phase retrieval problem in adaptive radar system. We numerically demonstrate improved performance in terms of speedier convergence and lower normalized mean squared error of using Zadoff Chu sequence compared to random initial seed alternating projection methods.

VI. APPENDIX

A. Monotonicity of E w.r.t. μ

Theorem: The Energy of the transmitted signal E is a monotonic function of the Lagrangian multiplier parameter μ.

Proof: From the energy constraint E on the transmitted radar signal, and expressing the same in terms of the water-illuminated waveform solution, we have

$$E = \int_{\Omega} |X_{SINR}(f)|^2 df$$
$$= \int_{\Omega} \frac{\sqrt{|H(f)|^2 S_{nn}(f)}}{S_{cc}(f)} [\mu - \frac{\sqrt{S_{nn}(f)}}{|H(f)|}]^+ df \quad (25)$$

Observe that

$$0 \le D_{min} \le \frac{\sqrt{S_{nn}(f)}}{|H(f)|} \le D_{max} \qquad \forall f \quad (26)$$

where $D_{min} = \min_{f \in \Omega} D(f)$ and $D_{max} = \max_{f \in \Omega} D(f)$ The region of support or limit of integration is defined by the set

$$\left\{ f : \frac{\sqrt{S_{nn}(f)}}{|H(f)|} < \mu \right\} \quad (27)$$

and the entire bandwidth is defined as

$$\left\{ f : \frac{\sqrt{S_{nn}(f)}}{|H(f)|} > \mu \right\} \cup \left\{ f : \frac{\sqrt{S_{nn}(f)}}{|H(f)|} \le \mu \right\} \quad (28)$$

It is worth noting here that,

$$\mu = D_{min} \Rightarrow E = 0 \quad (29)$$

and for $\mu = D_{max}$

$$\Rightarrow E = \int_{\Omega} \frac{|H(f)|\sqrt{S_{nn}(f)}}{S_{cc}(f)} \left(D_{max} - \frac{\sqrt{S_{nn}(f)}}{|H(f)|} \right) df \ge 0 \quad (30)$$

which means $E(D_{max}) > E(D_{min})$. To prove the monotonicity of E in μ, we observe that the integrand in the energy constraint equation is positive and is a function of μ and f. Now, we can argue that at each f, the integrand is a linearly increasing function of μ, and also the range of support

$$\{ f : \frac{\sqrt{S_{nn}(f)}}{|H(f)|} > \mu \}$$

increases with μ. Now since the integrand is positive for each f, integration (or summation) is going to increase the value of the integral. Hence, we conclude that E is a monotonically increasing function of μ. ∎

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