New Method for the Determination of Asteroid Shape using Spherical Segmentation Based Delay Doppler Analysis

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Abstract:

Conventional methods of Asteroid shape determination using Radar delay-Doppler image inversion involve making a polyhedral model with enough number of points. Obtaining an shape estimate is tedious in this process due to large number of parameters. The proposed spherical segmentation of any object involves relatively lesser number of parameters and has eigenfunction relationship with delay-Doppler operator. This expedites obtaining the first model for iterative improvements.

Keywords: Spherical Segmentation, Delay-Doppler, shape models

I INTRODUCTION

Radar is used in Astronomy to capture information about physical properties, orbits and rotational dynamics of near-earth Astronomical bodies which includes planets and Near-Earth Asteroids (NEAs) as well as few main belt asteroids. These objects are spatially overspread (10-100 order of magnitude) and hence give multiple reflections as transmitted pulse encounters surfaces of the object at different distances (ranges). Echo power measurement as a function of time delay (range) and Doppler frequency shift (radial velocity) gives a handle to produce 2D images of the objects. With longer observations, it is possible to reconstruct 3D structure of the object and rotation parameters.

II THE PROBLEM

It is important to determine the center of mass (COM) and thus shape of the NEAs to accurately predict their orbits, since it is important to differentiate between Doppler shift due to object's bulk motion and the intrinsic Doppler dispersion due to target rotation[1].In a delay-Doppler image, it is not known as to which point on the surface at a particular range contributes to echo power at a particular frequency. Hence it can be viewed as many to one mapping, leading to an inherent aliasing known as 'North/South ambiguity'.

Conventional method of reconstructing asteroid shape uses a polyhedral shape model with enough vertices to ensure reconstruction of the most detailed structure revealed in the images. Typically an objective function corresponding to deviation between observed delay-Doppler spectra and model-prediction is minimized.

$$\Phi(x) = \chi 2(x) + \Sigma\, \beta_i\gamma_i(x) \qquad (1)$$

where penalty function $\gamma i(x)$ has weight βi and $\chi 2(x)$ is the weighted sum of squared residuals. Certain features are suppressed using penalty functions[2].

Polyhedral model based shape prediction is computational complex due to enormous number of parameters (order N, where N is number of delay-Doppler cells). Also reconstruction becomes increasingly difficult with this method due to coupling between the spin-state parameters and the shape parameters in case of a Non-Principal Axis rotation of the body [1],[4]. This is mainly because of the nature of the coupling between the eight spin-state parameters and the shape parameters and inherent slow nature of NPA asteroids[1],[3],[6].

This paper will discuss the spherical segmentation based shape-prediction algorithm proposed by us. The method is supported by a number of representative simulations. The usefulness of the algorithm is discussed towards the end.

III PROPOSED METHOD

The proposed method uses spherical segmentation of any object, which relies on the assumption that every object can be modeled as combination of several spheres of different radii centered at different locations. This decomposition is analogous to discrete Fourier Transform. In this method, we plot echo-power(on z-axis) as a function of delay(Y-axis) and Doppler shift(X-axis). Echo power is given colour gradient according to its magnitude and is thus plotted on the 2D delay-Doppler plane with appropriate echo colour in each delay-Doppler cell as shown in fig 1. This representation gives striking results for shape and spin-state prediction using spherical segmentation.

1 The Algorithm

In any shape prediction algorithm, an initial model is taken and is simulated to generate the delay-Doppler spectrum. It is then iteratively improved. In this algorithm, inputs are taken in the form of a set of spheres with their relative locations and radii. The number spheres to be taken and the relative radii are estimated first from the observed delay-Doppler spectrum of the asteroid. The method to obtain this first estimate is described later. A feature has been developed to incorporate concave regions on the asteroid. Negative spheres are inserted by user for concave regions on object.

With this first estimate, the user defined object having user defined apparent axis of rotation is illuminated with appropriate pulse and the direct reflections are observed. The object is discretized into a 3D matrix of 40x40x40. Based on the given parameters, a surface velocity matrix is generated for all the surface points. The delay-Doppler array is constructed based on positions and calculated apparent velocities for the surface points. The apparent velocities are taken to correct for earth's rotation. The surface is assumed to be smooth relative to wavelength used and the albedo is assumed to be uniform. Here, we are only interested in shape and not the scale. These assumptions can be relaxed without significant changes in results. The next set of spheres for the model is considered based on deviation of predicted and actual delay-Doppler spectrum. This sequence is repeated.

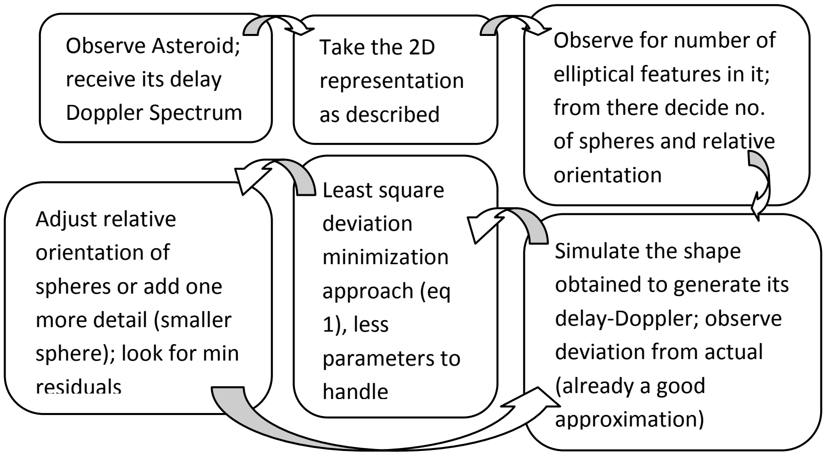

Figure 2: A flow chart describing the proposed shape prediction algorithm

2 Simulations

Various shapes were simulated using aforesaid strategy to generate the delay-Doppler spectrum and observe if it significantly correlates with the number of spheres to be accounted in the model. We started the exercise with 1 sphere, a combination of 2 spheres, 3 spheres and so on by also varying the axis of rotation both relative angle-wise and position-wise. A celebrated asteroid named '216 Kleopatra' is also simulated with this algorithm as a combination of 7 spheres. The results indicate a strong correlation between the number of spheres and the delay-Doppler spectrum of the object, making spherical segmentation very useful technique.

IV RESULTS

1 With 1 Sphere Central Axis

One sphere is given as input with certain angular velocity about an axis passing through center. It is illuminated and viewed from X-axis.Results shown in Fig.2.

2 With 1 Sphere Offset Axis

1 sphere is given as input with certain angular velocity about an axis offset by 5 units from center and at an angle 45^0. It is illuminated and viewed from Y-axis.Results shown in Fig.3.

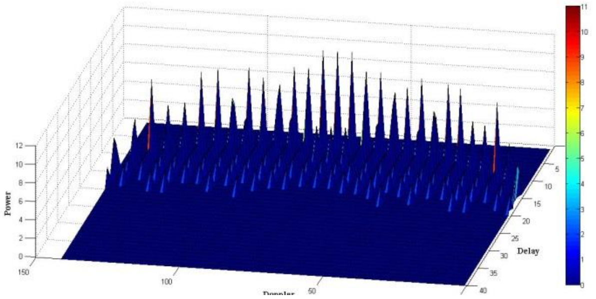

Figure 3a: The delay-Doppler Spectrum of a sphere(Central Axis rotation) in 3D

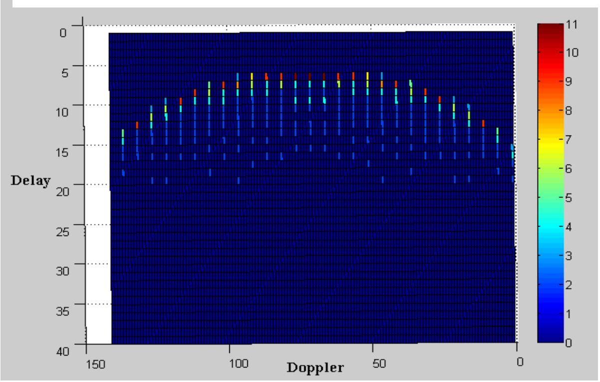

Figure 3b: The delay-Doppler Spectrum of a sphere(Central axis rotation) in 2D

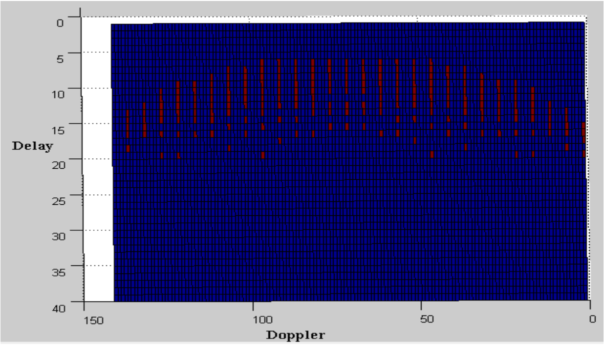

Figure 4: The delay-Doppler Spectrum of a sphere(Off-axis rotation) in 2D

3 *With 3 Spheres Arbitrary Axis*

A combination of 3 spheres is provided as input to the program. Various combinations have been tested. A comparison between two such combinations is shown. The axis of rotation used is the same in the two cases and view angle is also kept the same.

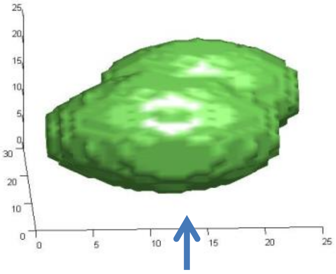

Figure 5a: A combination of 3 spheres, arrow indicating the line of observation

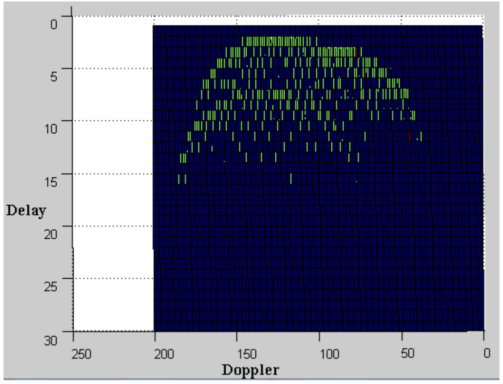

Figure 5b: Delay-Doppler spectrum of the combination (a) of 3 spheres

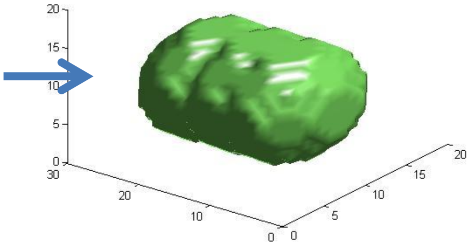

Figure 6a: A combination(b) of 3 spheres, arrow indicating the line of observation

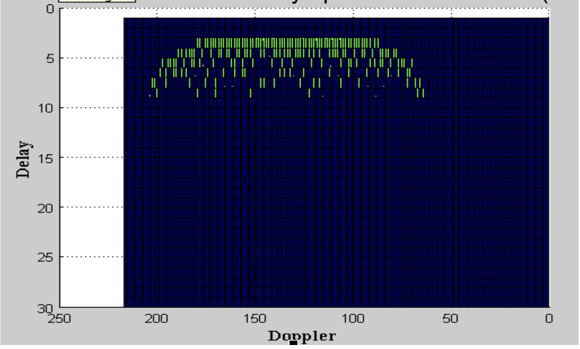

Figure 6b: Delay-Doppler spectrum of combination (b) of 3 spheres

4 *A sphere with a Concave Crater*

A sphere same as sphere for fig.1 is taken and a crater was incorporated using the negative-sphere feature. The signature of the crater will be seen when the asteroid turns and the crater cross section becomes lateral to the line of observation. One such observation is simulated and shown in Fig.6. It is seen that a left-right asymmetry is built up when the crater becomes lateral to line of sight. This asymmetry will not be there when crater has some component of its area facing towards earth, since the projected are remains the same in such condition

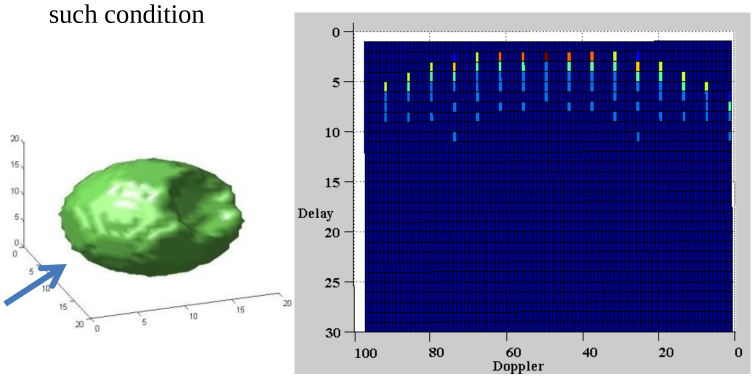

Figure 7: The sphere with crater and its delay-Doppler spectrum in 2D

5 *A Model of Asteroid 216 Kleopatra*

This celebrated asteroid has been extensively studies in Radar Astrometry. It has a dog-bone shape and we modeled it using a combination of 7 spheres. Note the striking similarity in the shape and in the delay-Doppler spectrum in 2D.

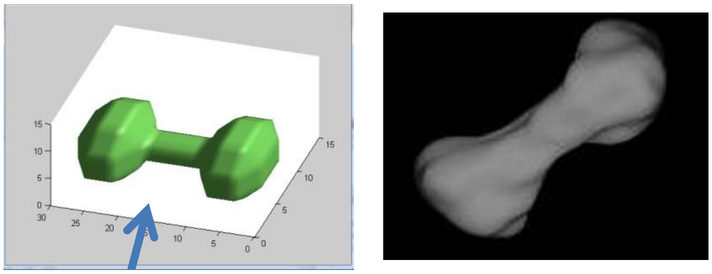

Figure 8a: Kleopatra Asteroid as- modeled by us and as -modeled in the literature [7]

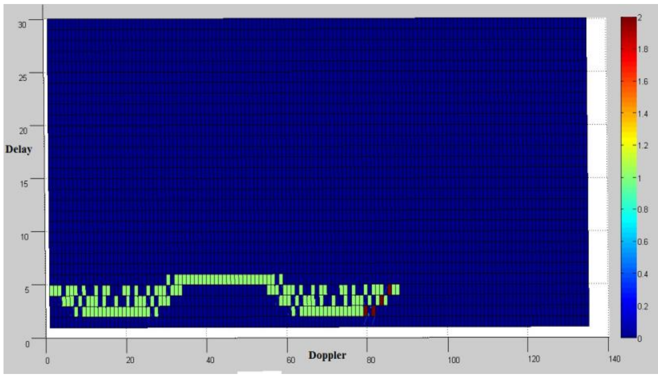

Figure 8b: Delay-Doppler spectrum of the as modeled asteroid

V OBSERVATIONS

There are some very important observations that can be made from the representative examples considered here. It has been found that the particular 2D representation that has been used here gives a very intuitive correlation with the number of spheres to be incorporated in the model as opposed to the non-intuitive 3D representation, where such conclusions are difficult to make. It can be clearly observed that the number of spheres to be considered and their relative orientations and sizes can be estimated from this representation. Hence this particular representation of the delay-Doppler spectrum is easy to invert to obtain the coarse estimate of the shape of asteroid in terms of spherical segments. It can be said that a sphere is an eignefunction for the delay-Doppler operator in this particular representation. Thus the spheres appear as ellipses in delay-Doppler spectrum and hence, shape prediction is facilitated by spherical segmentation.

Also, it can be noted that the axis of rotation has almost no effect on the nature of delay-Doppler spectrum in this representation. Thus we can also say that the problem of spin vector coupling with shape is no longer encountered. The spin state, in this strategy can be obtained by observing a few complete rotations of the asteroid.

In the conventional polyhedral model approach, it is difficult to converge quickly towards a coarse estimate since the number of parameters is inherently large, whereas this method is computationally less complex and thus quicker. After a coarse estimate, the conventional ‘Penalty-function objective minimization’ (as described previously) can easily be applied since the number of parameters is drastically reduced. Axis of rotation can be determined by observing extended time variability of echo power distribution (and thereby mass-distribution) in Doppler shifts. Precession motions can also be incorporated as another parameter in the penalty-function.

Asteroid-like objects generally give very small window of time(around a day) when they are closest to earth and thus giving best Radar visibility. These events are also rare. Additionally, there are uncertainties in ephemeris based on optical data[1]. Typically they are around ~15-arcmin. To ensure reasonable sensitivity, pointing should be good to at least 15 arcsec[1].

Once detection takes place, position of center of mass(COM) has to be quickly determined. Thus, orbit refinement is tightly coupled to determination of shape and physical properties. This fast shape predictor algorithm will enable the Radar Telescope(Arecibo/Goldstone) to quickly calculate approximate position of COM of the object and track it.

CONCLUSIONS

A proof of concept for the spherical segmentation based shape prediction algorithm has been produced in the paper. Features like concave craters can also be detected by looking for diminishing power in the Doppler extremities of the spectrum as a function of time. The concave craters can be thought of as negative spheres. The combination of two spheres generated a delay-Doppler spectrum showing two elliptical regions. Similarly combination of a sphere and a negative sphere will appear as delay-Doppler spectrum of one sphere subtracted from the other when that crater becomeslateral to the line of sight as the asteroid rotates. Hence, we can say that a sphere is found to be an eigenfunction of the delay-Doppler operator on a shape, thus making it possible to apply principal of superposition. This completely simplifies the process of shape detection. More number of discrete cells can be used to improve the quality of estimate.

It is possible to relax the assumptions of uniform surface roughness and albedo without sacrificing the utility of the algorithm, since it relies on the 2D representation of the delay-Doppler spectrum and is not much affected by absolute value of the power received. The absolute power will be helpful in characterizing surface of the asteroid. Thus we have separated out the explicit shape-relevant information from the entire collection of the data received, which makes the algorithm simpler and faster.

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