Simulation of Digital Beam Former & Implementation on XILINX FPGA

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ABSTRACT

The project deals with the approaches for implementation of a Fixed weight Digital Beamformer & an adaptive Beamformer using QRD-RLS (Q R decomposition based Recursive Least Squares) algorithm for Phased Array Radar applications. The digital beamformer architecture includes the complex operations such as down conversion which is done in parallel for the signal coming from each of the antenna elements and the filtering. The digital baseband echo signal from each of the antenna element is multiplied by the complex weights and then summed to form the beam. In case of Fixed Beamformer architecture, the weights are fixed according to the angle of beam required. QRD-RLS algorithm is suitable for parallel & pipelined implementation thus making it very useful in applications where speed, accuracy & numerical stability are of utmost importance- such as Phased Array Radar receiver. The high performance FPGA Xilinx Virtex-6 & Kintex-7 Boards are employed to implement the Fixed & Adaptive Beamformer architectures respectively. The complex weights for adaptive beamforming are calculated adaptively using highly stable Q-R decomposition and Inverse Q-R decomposition based recursive least squares algorithm. The architecture used for implementation of QRD-RLS algorithm in a parallel & pipelined manner is called Systolic Array.

Conventional methods of implementation of beamforming make the system cumbersome and sensitive to temperature and other unavoidable environmental conditions. FPGA based implementation finds huge applications in modern radars as this implementation makes the system immune to the limitations that the analog methods face. At the same time, the proposed beamforming system enjoys advantages of a reconfigurable design and low cost.

Key words: Beamformer, QRD-RLS, Phased Array, Virtex, Kintex, Systolic

1.0 Introduction

Systems such as radar receiver which are designed to receive spatially propagating signals often encounter the presence of interference signals. If the desired signal and interference occupy the same frequency band, then temporal filtering cannot be used to separate signal from interference. However, the desired and interfering signal usually originate from different spatial locations. This spatial separation can be exploited to separate signal from interference using spatial filter at the receiver. Implementing a spatial filter requires processing of data collected over a spatial aperture. When the spatial sampling is discrete, the hardware that performs spatial filtering is termed as beam former. Digital beam forming consists of the spatial filtering of a signal where the phase shifting, amplitude scaling, and adding are implemented digitally. The idea is to use a computational and programmable environment which processes a signal in the digital domain to control the progressive
phase shift between each antenna element in the array. Digital beam formers can accomplish minimization of side-lobe levels, interference cancelling and multiple beam operation without changing the physical architecture of the phased array antenna. Adaptive filters for phased array radar applications distinguish the properties of signals and noise through spatial filtering, where an array of independent sensors provide a means of sampling the received signal in space. The sensor outputs are then processed by a transversal filter to produce the beam at the output. The primary purpose of the adaptive filter here is to protect a target signal while cancelling an interference signal. QRD-RLS algorithm is a good option in such applications where speed of convergence is of paramount importance and an efficient, reliable, and numerically robust adaptive filter is needed.

The problem of designing efficient adaptive filter algorithm is an important part of general radar and communication systems. The performance of the conventional signal reception system is sensitive to decrease in signal to noise ratio caused by undesired interference signal which enter the system either by the beam pattern side lobes or main lobe. As radar traffic increases the suppression of interference becomes more important in all applications. Adaptive filter algorithm can be used in a radar array system to preserve desired signal in the presence of interference signal. Moreover, the adaptive filter gives the ability to the antenna array system to automatically sense the presence of the interference signals and to suppress precisely these interference signals while simultaneously enhancing the desired signal.

2.0 Objective
Digital beam forming has many of the advantages as digital computational environment has over its analog counterpart. In most cases, less power is needed to perform the beam steering of the phased array antenna. Another advantage is the reduction of variations associated with time, temperature, and other environmental changes found in analog devices. Digital beam formers can accomplish minimization of side-lobe levels, interference cancelling and multiple beam operation without changing the physical architecture of the phased array antenna. Every mode of operation of the digital beam former is created and controlled by means of code written on a programmable device of the digital beam former, in this case a Xilinx FPGA. Designing a beam former is an important part of general radar and communication systems. There are several methods of beamforming being used currently. The performance of the conventional signal reception system is sensitive to decrease in signal to noise ratio caused by undesired interference signal which enter the system either by the beam pattern side lobes or main lobe. The signal processor does the required processing for the received signals and helps to form the beams in specific directions. Now a day, for efficient beamforming, adaptive beamforming is used. Adaptive filter algorithm can be used in a radar array system to preserve desired signal in the presence of interference signal. As radar traffic increases the suppression of interference becomes more important in all applications, where these adaptive beamforming finds its main applications. Adaptive filters for phased array radar applications distinguish the properties of signals and noise through spatial filtering, where an array of independent sensors provide a means of sampling the received signal in space. The sensor outputs are then processed by a transversal filter to produce the beam at the output. The primary purpose of the adaptive filter here is to protect a
target signal while cancelling an interference signal. Moreover, the adaptive filter gives the ability to the antenna array system to automatically sense the presence of the interference signals and to suppress precisely these interference signals while simultaneously enhancing the desired signal. QRD-RLS algorithm is a good option in such applications where speed of convergence is of paramount importance and an efficient, reliable, and numerically robust adaptive filter is needed.

2.1 Problem Statement
The traditional analog way to perform beamforming is very expensive, and it is sensitive to component tolerances and drifts, while modern technology offers high speed A/D converters and Digital Down Converters (DDCs), fundamental blocks for digital beamforming. The conventional Recursive Least Square (RLS) algorithm replaces the gradient step size in conventional Least Mean Square (LMS) algorithm by the gain matrix. The RLS algorithm offers better convergence rate, steady-state Mean Square Error (MSE), and the parameter tracking capability over the LMS-based algorithms. However its complexity is proportional to the squared number of the unknown system parameters. To overcome the drawback of conventional RLS algorithm, a numerically stable RLS algorithm, called the QR decomposition RLS (QRD-RLS) is proposed.

3.0 System Overview
QRD-RLS algorithm is used to remedy the problems of ill conditioning and VLSI implementation, which is based on orthogonalization techniques such as Givens Rotation, Householder Transformation, and Gram-Schmidt Technique etc. In this algorithm, the QR decomposition of the input data matrix is computed using Givens rotation and the least squares (LSs) weight vector is solved by back substitution. The back substitution however is a costly operation to be performed in the array structure. So an inverse QRD-RLS algorithm was proposed where the LS weight vector is computed without the back substitution. The algorithm, which has better convergence performance than RLS algorithm in the presence of the strong look-direction signal, is the improved LMS.

The paper illustrates the implementation of QRD-RLS algorithm for adaptive filter using Givens Rotation orthogonalization technique. QR based algorithm directly deals with observed data matrix and achieves the requirements of computational efficiency, robust numerical stability and VLSI parallel and pipelined architecture implementation. Systolic array is an architecture on which parallel algorithms can run. A systolic array is an arrangement of processors in an array where data flows synchronously between neighbours across the array, usually with different data flowing in different directions. Processors perform a sequence of operations on data that flows between them and operate concurrently. Each processor at each step takes in data from one or more neighbours, processes it and, in the next step, outputs results in the opposite direction.

3.1 Digital Beamforming
In beam forming, both the amplitude and phase of each antenna element are controlled. The operations of phase shift and amplitude scaling for each antenna element, and summation for receiving, are done digitally. Either general-purpose DSP’s or dedicated beam forming chips are used. Digital processing requires that the signal from each antenna element is digitized using an ADC. Since radio signals above shortwave frequencies (>30 MHz) are too high to be directly digitized at a reasonable cost, digital beam forming receivers use
analog “RF translators” to shift the signal frequency down before the ADC. Once the antenna signals have been digitized, they are passed to “digital downconverters” that shift the radio channel’s centre frequency down to 0 Hz and pass only the bandwidth required for one channel. Fig.1 shows a complete digital beam forming receiver. One set of antenna elements, RF translators, and ADC can be shared by a number of beam formers. All RF translators and ADC converters share common oscillators so that they all produce identical phase shifts of the signal. Within the digital beam former, all digital downconverters share a common clock, are set for the same centre frequency and bandwidth, and their digital local oscillators are in-phase so that all phase shifts are identical. Each DDC’s baseband output is multiplied by the complex weight for its antenna element, and the results are summed to produce one baseband signal with directional properties. A demodulator would then follow to recover information from the radio signal.

3.1 Advantages of Digital Beamforming

DigitalBeamforming(DBF) offers advantages in terms of power consumption, flexibility and accuracy. Furthermore advantages of DBF implementation can be seen in providing a high flexibility system using advanced adaptive algorithms, for instance providing null steering, accurate main beam, superresolution, array element pattern correction, control of side-lobe levels, self calibration, multiple beam operation, and radar power and time management without changing the physical architecture of phased array antenna. Every mode of operation of digital beamformer is created and controlled by means of code written on a programmable device.

4.0 Conventional QRD-RLS Algorithm

The main advantages associated to the QRdecomposition RLS (QRD-RLS) algorithms, as opposed to their conventional RLS counterpart, are the possibility of implementation in systolic arrays and the improved numerical behaviour in limited precision environment. Conventional QRD-RLS implementation involves converting the
input data matrix to the upper triangular matrix using QR-Decomposition technique in pipelined manner and then performing back substitution to generate the weights. The back substitution procedure is essentially a non-pipelined procedure because the calculation of weights starts by using the last row of upper triangular data matrix whereas the first row of the data matrix gets updated first when new data arrives. Thus the disadvantage with back substitution process is that it takes more time for calculation of weights till the last row of data matrix gets updated. Therefore back substitution takes more time to generate weights when compared to Inverse QRD-RLS implementation where weights are calculated in pipelined manner, where the first row gets updated only at the end, but it gets the input data first as shown in Fig.2.

QR matrix decomposition (QRD) is also known as orthogonal matrix triangularization is nothing but decomposition of matrix (B) into an orthogonal matrix (Q) and an upper triangular matrix (R). The equation can be given by \( By = X \) …… 5.1

Here, B, Y and X are matrices, B is of the order \( N \times N \), Y and X are a column vectors whose order is \( N \times 1 \), B and X are known and Y is unknown.

The aim is now to determine the N different unknown variables in the matrix Y. So substituting QR for B gives us:

\((QR)Y = X \) .... 5.2

which gives: \( RY = Q^{-1}X \) ... 5.3

Q is an orthogonal matrix and \( Q^{-1} \) is the complex conjugate transpose of Q, this requires only few resources to perform when implemented in hardware.

\( RY = X' \) .... 5.4

Where: \( X' = Q^{-1}X \) .... 5.5

The weight calculation is a time consumption process because it uses back substitution process for the calculation of weight. It needs all the rows to get updated in order to calculate the weight using conventional QRD. This is the main drawback in conventional QRD. An alternate method is to implement inverse QRD-RLS to calculate the weight in which weights are calculated in a parallel and pipelined manner.

4.1 Inverse QRD-RLS Algorithm

An alternative approach to the conventional QRD-RLS algorithm based on the update of the inverse Cholesky factor known as the Inverse QR decomposition (IQRD-RLS) algorithm, allows the calculation of the weight vector without back-substitution. This algorithm, besides the inherited numerical robustness of its family, provides the coefficient vector at every iteration, without having to resort to the computationally onerous backward or forward substitution procedures. Conventional QRD-RLS update scheme involves two computational steps which cannot be efficiently combined on a pipelined array. To circumvent the difficulty, inverse updating-based algorithms are developed.

5.0 Design Implementation

Fig.3 shows the top level architecture of 8 channels Fixed Beamformer implemented. The phase in for each channels to be generated are initially stored in registers and are entered through a GUI (Graphical User Interface) for DDS (Direct Digital Synthesizer). The outputs from DDS are two sinusoidal waveforms (sine and cosine) with respect to the phase in given. The DDS generate outputs for each channel in
a time-multiplexed fashion. The output for a particular channel will be given every 8-cycles. The outputs for channel 0 are given first. The CHANNEL output can be enabled through the GUI to provide the index of the channel at the output. These time-multiplexed signals are next time de-multiplexed for the parallel operation on each separate channel output.

Fig.3 Top Level Architecture of FPGA Implementation of a 8 channel Fixed Digital Beamformer

The quadrature baseband i and q components can be represented as

\[ s(t) = x(t) + j y(t) \]  \hspace{1cm} (7.1)

\( s(t) \) is the complex baseband signal
\( x(t) = i(t) \) the real part represented as I
\( y(t) = -q(t) \) the imaginary part as Q

For beamforming, the complex baseband signals are multiplied by the complex weights to apply the phase shift and amplitude scaling required for each antenna element.

\[ w_k = a_k \begin{bmatrix} \cos(\theta_k) \\ \sin(\theta_k) \end{bmatrix} \]  \hspace{1cm} (7.2)

\[ w_k = a_k \cos(\theta_k) + j a_k \sin(\theta_k) \]  \hspace{1cm} (7.3)

\( w_k \) is complex weight for the \( k \)th antenna element
\( a_k \) is the relative amplitude of the weight and \( \theta_k \) is the phase shift of the weight.

Since the implementation is Fixed Beamformer the weights for all 8 elements are same i.e., fixed. The \( \theta \) value to generate fixed weights is given through a GUI and is stored in a register. The weights are generated according to the \( \theta \) value given and are complex multiplied with the complex baseband signal \( s(t) \).

A general-purpose DSP can implement the complex multiplication for each antenna element:

\[ Y(t) = s_k(t) w_k = a_k \begin{bmatrix} x_k(t) \cos(\theta_k) - y_k(t) \sin(\theta_k) \\ x_k(t) \sin(\theta_k) + y_k(t) \cos(\theta_k) \end{bmatrix} + j \begin{bmatrix} x_k(t) \sin(\theta_k) + y_k(t) \cos(\theta_k) \end{bmatrix} \]  \hspace{1cm} (7.4)

\[ Y(t) = \text{Beam}(\text{real}) + \text{Beam}(\text{imaginary}) \]  \hspace{1cm} (7.5)

where \( \text{Beam}(\text{real}) = a_k [x_k(t) \cos(\theta_k) - y_k(t) \sin(\theta_k)] \) represented as BR in Fig.7.1.

\( \text{Beam}(\text{imaginary}) = a_k [x_k(t) \sin(\theta_k) + y_k(t) \cos(\theta_k)] \) represented as BI in Fig.7.1.

5.1 Adaptive Beamforming using Inverse QRD-RLS Algorithm

5.1.1 Systolic Array Implementation

The systolic array implementation of a given algorithm consists of mapping the algorithm in a pipelined sequence of basic computation cells. These basic cells perform their task in parallel, such that in each clock period all the cells are activated.

A Givens rotation requires two basic steps. The first step is the calculation of the sine and cosine which are the elements of the rotation matrix. The second step is the application of the rotation matrix to given data. Therefore, the basic computational elements required to perform the systolic array implementation of the QR-RLS algorithm are the angle and the rotation processors. The angle processor computes the cosine and sine, the rotation processor performs the rotation between the data coming from input with the internal element of the matrix and transfers the result to output.

Here the systolic array is used for the implementation of Givens rotation. A Givens rotation requires two basic steps. The first step is the calculation of the sine and cosine which are the elements of the rotation matrix. The second step is the application of the rotation matrix to given data. Therefore, the basic computational elements required to perform the systolic array implementation of the QR-RLS algorithm are the angle and the rotation matrix.
processors. The angle processor computes the cosine and sine, transferring the results to outputs 1 and 2 respectively, whereas in output 3 the cell delivers a partial product of cosines meant to generate the error signal. The rotation processor performs the rotation between the data coming from input 1 with the internal element of the matrix $U(l)$ and transfers the result to output 3. This processor also updates the elements of $U(l)$ and transfers the cosine and sine values to the neighboring cell on the left.

5.2 Inverse QRD-RLS Implementation
Inverse QR decomposition (IQRD-RLS) algorithm, allows the calculation of the weight vector without back-substitution. This algorithm, besides the inherited numerical robustness of its family, provide the coefficient vector at every iteration, without having to resort to the computationally onerous backward or forward substitution procedures. Conventional QRD-RLS update scheme involves two computational steps which cannot be efficiently combined on a pipelined array. To circumvent the difficulty, inverse updating-based algorithms are developed.

6.0 Hardware and Software Requirements
6.1 Hardware:
- Virtex-6 FPGA based custom board (For Fixed Beamforming).
- Kintex-7 FPGA (For Adaptive Algorithm).
- Virtex board for interfacing and testing.

6.2 Software:
- Chip-scope tool.
- MatLab.

Fig. 4 Dataflow diagram weight vector calculation using inverse QRD-RLS

7.0 SIMULATION AND RESULTS
7.1 Matlab Results
7.1.1 Fixed Beamforming
1. Matlab Simulation for Linear Array with 8 elements and $\theta = 60^\circ$
Fig. 5 Linear Array Beamforming Pattern (dB plot) for $\theta = 60^\circ$ & $N = 8$

Fig. 6 Linear Array Beamforming Pattern (dB plot) for $\theta = 60^\circ$ & $N = 16$

Fig. 7 Linear Array Beamforming Pattern (dB plot) for $\theta = 60^\circ$ & $N = 1024$

The MatLab simulation results of Fixed Digital Beamforming for different Linear Array configurations with element spacing $d=0.5\lambda$ are shown in Fig. 5 to 7. It can be seen that as the number of elements in the array increases, the directivity and gain of the main beam increases i.e. the main beam gets narrower as the $N$ increases. Also as the number of elements increases, the side lobes become smaller.

7.1.2 Adaptive Beamforming

1. Matlab Simulation for Linear Array with 8 elements and $\theta = 30^\circ$ using LMS algorithm. Angle of Interference = $50^\circ$

2. Matlab Simulation for 8×8 Planar Array with $\theta = 60^\circ$ and phi = $80^\circ$ using LMS algorithm. Angle of Interference with $\theta = 100^\circ$ and phi = $120^\circ$

3. Following is the result of the implementation of adaptive beamforming using conventional QRD-RLS algorithm for 16 element linear phased array radar in MATLAB.

Input angle of arrival for desired signal=40°

Input angle of arrival for interference signal=20°

Fig. 8 Linear Array Beamforming Pattern using LMS algorithm (dB plot) for $\theta = 30^\circ$

Fig. 9 Planar Array Beamforming pattern using LMS algorithm with $\theta = 60^\circ$ and phi = $80^\circ$

Fig. 10 N=16,Weights QRD-RLS in Matlab
4. Following is the result of the implementation of adaptive beamforming using inverse QRD-RLS algorithm for 16 element linear phased array radar in MATLAB. 
Input angle of arrival for desired signal = 25°
Input angle of arrival for interference signal = 10°

Fig.11 Beam plot for the 16 element linear antenna array using weights generated by inverse QRD-RLS in Matlab

5. Following is the result of the implementation of adaptive beamforming using inverse QRD-RLS algorithm for 16 element linear phased array radar in MATLAB.

Input angle of arrival for desired signal = 0°
and for interference signal = 40°
Number of beams = 9

Fig.12 Linear array of sixteen elements 9 adaptive beams

6. Following are the results of the implementation of adaptive beamforming using inverse QRD-RLS algorithm for 16 element planar phased array radar in MATLAB.
Input azimuth angle = 0°
Input elevation angle = 10°

Fig.13 Beam plot for the 16 element planar antenna array using weights generated by inverse QRD-RLS in Matlab

7. Following are the results of the implementation of adaptive beamforming using conventional QRD-RLS algorithm for 16 element planar phased array radar in MATLAB.
Input azimuth angle = 40°
Input elevation angle = 20°

Fig.15 Beam plot for the 16 element planar antenna array using weights generated by conventional QRD-RLS in Matlab

7.2 Hardware Results

7.2.1 Resource Utilization
The resource utilization for Conventional QRD-RLS algorithm is less as compared to
Inverse QRD-RLS algorithm. Conventional QRD-RLS implementation involves converting the input data matrix to the upper triangular matrix using QR-Decomposition technique in pipelined manner and then performing back substitution to generate the weights. The disadvantage with back substitution process is that we have to wait for calculation of weights till the last row of data matrix gets updated. Therefore back substitution takes more time to generate weights when compared to Inverse QRD-RLS implementation where weights are calculated in pipelined manner.

8. Conclusion and Future Scope

An optimum architecture of Fixed Digital Beamformer & highly computationally efficient adaptive algorithms- conventional and inverse QRD-RLS are being implemented. It is a systolic implementation where optimal weights are calculated. The algorithm has a modular design where the few modules are reused for the complete architecture implementation. This algorithm is more stable than other adaptive algorithms like LMS & RLS. It is also suitable for parallel and pipelined process thus making its implementation useful where speed and accuracy is more important. The Fixed Beamformer architecture is implemented on Virtex-6 FPGA & QRD-RLS algorithm is implemented on a highly efficient Kintex-7 FPGA board. The use of FPGAs has enabled a reduction in the resource utilization compared to the analog versions. The QRD-RLS algorithm for 4 element linear array can be used as the component to implement the filter for large phased array antennas. The design enjoys advantages like reconfiguration and also low cost. The next step for enhancing the DBF system is including online weight calculation algorithm such as QRD-RLS, inside the FPGA, so as to enable radar to track the changes in continuously varying environment, thus making the DBF system robust and efficient. Depending on the availability of the resources and the input sample rate- conventional or the inverse QRD-RLS algorithm can be selected. The same work can be extended for a planar array of larger array dimensions.

9. References


NIMHANS Convention Centre, Bangalore INDIA
[10] Digital beamforming in wireless communications

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**Dhiraj Malode** obtained his B.E. from HMSIT, Tumkur and M.Tech from PESIT both affiliated to VTU, Belgaum. He was awarded Best Outgoing Student in HMSIT, and secured second rank in M.Tech first year. He worked on a project on Novel Area-Efficient FPGA Architecture for FIR Filtering With Symmetric Signal Extension and FPGA Based Error Detection and Correction Using Orthogonal Code and Closest Match Technique and on Beamforming Network. His interest is on the hardware implementation of systems.

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